Density fluctuations and compressibility matrix for population or mass imbalanced Fermi-Fermi mixtures

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(Dated: October 19, 2011)

We describe the relation between the isothermal atomic compressibility and density fluctuations in mixtures of two-component fermions with population or mass imbalance. We derive a generalized version of the fluctuation-dissipation theorem which is valid for both balanced and imbalanced Fermi-Fermi mixtures. Furthermore, we show that the compressibility, its critical exponents, and phase boundaries can be extracted via an analysis of the density fluctuations as a function of population imbalance, interaction parameter or temperature. Lastly, we demonstrate that in the presence of trapping potentials, the local compressibility and local density-density correlations can be extracted via a generalized fluctuation-dissipation theorem valid within the local density approximation.

PACS numbers: 03.75.Ss, 03.75.Hh, 05.30.Fk

Very recent experimental advances in Bose and Fermi systems have allowed for studies of density fluctuations and the use of the fluctuation-dissipation theorem to obtain information about some thermodynamic properties of ultra-cold atoms. In the fermion case, the measurement of the density fluctuations and the atomic compressibility was extracted for non-interacting three-dimensional systems in harmonic traps [1, 2], while in the boson case, the connection between density fluctuations and compressibility was used to study superfluidity in a two-dimensional system, and extract critical exponents associated with the transition from a normal Bose gas to a Berezinskii-Kosterlitz-Thouless superfluid [3]. The experimental extraction of the isothermal compressibility from density-fluctuation measurements were suggested several years ago both in harmonically confined systems [4] and optical lattices [5], but only recently improvements in the detection schemes of density fluctuations became sufficiently sensitive to extract this information from experimental data [1–3].

In principle, there are no major technical impediments to use the same technique for the study of density fluctuations in population imbalanced Fermi-Fermi mixtures of equal masses [6, 7] or unequal masses [8], where the compressibility and spin susceptibility matrix elements could be directly extracted from the density and density fluctuation profiles as discussed below.

In this manuscript, we show that some local and global thermodynamic properties can be extracted through local measurements of densities and density fluctuations. We derive a generalized version of the fluctuation-dissipation theorem valid for any mixtures of atoms, and use it to analyze density fluctuations and the compressibility of mixtures of two-component fermions with and without population imbalance at low temperatures. For spatially uniform systems, we show that the global compressibility, phase boundaries and critical exponents can be extracted from measurements of the density and density fluctuations for each component as a function of population imbalance, interaction parameter or temperature. While for spatially non-uniform systems, we show that the local and global compressibility can be extracted from measurements of the local density and local density fluctuations for each component.

Hamiltonian: To investigate the physics described above, we start with the real space Hamiltonian ($\hbar = 1$) density for three dimensional s-wave superfluids

$$\mathcal{H}(\mathbf{r}) = \sum_{\alpha} \psi^{\dagger}_{\alpha}(\mathbf{r}) \left(-\frac{\nabla^2}{2m_\alpha} - \mu_\alpha + V_\alpha(\mathbf{r}) \right) \psi_\alpha(\mathbf{r}) + \hat{U}(\mathbf{r}), \tag{1}$$

where $\hat{U}(\mathbf{r}) = \int d\mathbf{r}' V_{\rm int}(\mathbf{r},\mathbf{r}') \psi^{\dagger}_{\uparrow}(\mathbf{r}') \psi^{\dagger}_{\downarrow}(\mathbf{r}') \psi_{\downarrow}(\mathbf{r}) \psi_{\uparrow}(\mathbf{r})$ contains the interaction potential $V_{\rm int}(\mathbf{r},\mathbf{r}') = -g\delta(\mathbf{r}-\mathbf{r}')$, and $\psi^{\dagger}_{\alpha}(\mathbf{r})$ creates fermions of mass m_α labeled by index α. In addition, $V_\alpha(\mathbf{r})$ and μ_α represent the trapping potential and the chemical potential for each fermion type, respectively. With the Hamiltonian $H = \int d\mathbf{r} \mathcal{H}(\mathbf{r})$, we can study Fermi systems of equal masses $m_\uparrow = m_\downarrow = m$ with population imbalance, or more generally we can study mixtures of fermions of unequal masses $m_\uparrow \neq m_\downarrow$.

From the grand partition function $Z = {\rm Tr} e^{-H/T}$, we can write the thermodynamic potential $\Omega = -T \ln Z$. First, we will ignore the trapping potential $V_\alpha(\mathbf{r})$ and discuss the spatially homogeneous case to simplify the discussion, but will return to the spatially inhomogeneous situation later. We set $V_\alpha(\mathbf{r}) = 0$ in Eq. (1) and rewrite the Hamiltonian H as $H_1 - \sum_\alpha \mu_\alpha \hat{N}_\alpha$, where $H_1 = \int d\mathbf{r} \left[\sum_\alpha \psi^{\dagger}_{\alpha}(\mathbf{r}) \left(-\frac{\nabla^2}{2m_\alpha} \right) \psi_\alpha(\mathbf{r}) + \hat{U}(\mathbf{r}) \right]$, and $\hat{N}_\alpha = \int d\mathbf{r} \psi^{\dagger}_{\alpha}(\mathbf{r}) \psi_\alpha(\mathbf{r})$ is the number operator for hyperfine state α. The average number of particles $N_\alpha = \langle \hat{N}_\alpha \rangle$ in hyperfine state α, defined by the thermodynamic average $\langle \hat{N}_\alpha \rangle = Z^{-1} {\rm Tr} \left[\hat{N}_\alpha e^{-H/T} \right]$, can be rewritten in terms of the thermodynamic potential as $N_\alpha = \langle \hat{N}_\alpha \rangle = -\partial \Omega / \partial \mu_\alpha |_T$.

Pseudo-compressibility matrix: Next, we define the

pseudo-compressibility matrix as

$$\tilde{\kappa}_{\alpha\beta} = T \frac{\partial N_\alpha}{\partial \mu_\beta}\Big|_T = -T \frac{\partial^2 \Omega}{\partial \mu_\alpha \partial \mu_\beta}\Big|_T, \qquad (2)$$

which through derivatives of Z can be expressed as the thermodynamic average $\tilde{\kappa}_{\alpha\beta} = \langle \hat{N}_\alpha \hat{N}_\beta \rangle - \langle \hat{N}_\alpha \rangle \langle \hat{N}_\beta \rangle$. The mechanical stability of the system is guaranteed when both eigenvalues of $\tilde{\kappa}_{\alpha\beta}$ are positive definite. Furthermore, $\tilde{\kappa}_{\alpha\beta}$ is a measure of density-density fluctuations:

$$\tilde{\kappa}_{\alpha\beta} = \langle (\hat{N}_\alpha - N_\alpha)(\hat{N}_\beta - N_\beta) \rangle. \qquad (3)$$

The corresponding generalized compressibility matrix $\kappa_{\alpha\beta}$ can be obtained from $\tilde{\kappa}_{\alpha\beta}$ through the relation $\kappa_{\alpha\beta} = \tilde{\kappa}_{\alpha\beta} / \left[\langle \hat{N}_\alpha \rangle \langle \hat{N}_\beta \rangle \right]$, describing a generalized fluctuation-dissipation theorem for multicomponent fermions.

Using similar experimental techniques to those described in Refs. [1, 2], it may be possible to measure the matrix elements of $\tilde{\kappa}_{\alpha\beta}$ directly. Thus, it is important to identify the relation between the isothermal compressibility $\kappa_T^{-1} = -V(\partial P/\partial V)|_T$ and the elements of the compressibility matrix $\kappa_{\alpha\beta}$.

Isothermal compressibility: The relation between κ_T and $\kappa_{\alpha\beta}$ can be established by recalling that the thermodynamic potential $\Omega = -PV$, where P is the pressure and V is the volume of the system. Defining $G = \Omega + PV = 0$, and recalling that Ω is a function of temperature T, volume V and chemical potentials μ_α results in $dG = -SdT - N_\uparrow d\mu_\uparrow - N_\downarrow d\mu_\downarrow + VdP = 0$. At constant temperature $dT = 0$, we can establish the relation $-VdP|_T = N_\uparrow d\mu_\uparrow|_T + N_\downarrow d\mu_\downarrow|_T$.

This means that the inverse isothermal compressibility $\kappa_T^{-1} = -V(\partial P/\partial V)_T$ can be writen in terms of isothermal partial derivatives of μ_α with respect to volume $\kappa_T^{-1} = N_\uparrow(\partial \mu_\uparrow/\partial V)|_T + N_\downarrow(\partial \mu_\downarrow/\partial V)|_T$. But in turn the partial derivatives $\partial \mu_\alpha/\partial V|_T$ can be expressed in terms $\partial \mu_\alpha/\partial N_\beta|_T$ and N_β, leading to

$$\frac{1}{\kappa_T} = \frac{T}{V} \left[\frac{N_\uparrow^2}{\tilde{\kappa}_{\uparrow\uparrow}} + \frac{N_\uparrow N_\downarrow}{\tilde{\kappa}_{\uparrow\downarrow}} + \frac{N_\downarrow N_\uparrow}{\tilde{\kappa}_{\downarrow\uparrow}} + \frac{N_\downarrow^2}{\tilde{\kappa}_{\downarrow\downarrow}} \right]. \qquad (4)$$

This expression can be written in the compact form $VT^{-1}\kappa_T^{-1} = \sum_{\alpha\beta} [\kappa_{\alpha\beta}]^{-1}$, by using the definition $\kappa_{\alpha\beta} = \tilde{\kappa}_{\alpha\beta}/(N_\alpha N_\beta)$. Therefore direct measurements of $\kappa_{\alpha\beta}$ lead to the isothermal compressibility κ_T of the system.

Connection to pseudo-spin susceptibility: We can also work with the total number of particles $N_+ = N_\uparrow + N_\downarrow$, the particle number difference $N_- = N_\uparrow - N_\downarrow$, and their corresponding chemical potentials $\mu_\pm = (\mu_\uparrow \pm \mu_\downarrow)/2$, respectively. In this case, we can define a similar pseudo-compressibility tensor $\tilde{\kappa}_{ij} = T\partial \langle \hat{N}_i \rangle / \partial \mu_j|_T = -T\partial^2 \Omega / \partial \mu_i \partial \mu_j|_T$, leading to $\tilde{\kappa}_{ij} = \langle \hat{N}_i \hat{N}_j \rangle - \langle \hat{N}_i \rangle \langle \hat{N}_j \rangle$, where the indices i, j can each take $\pm$ values. The corresponding expression for κ_T has exactly the same form as before:

$$\frac{1}{\kappa_T} = \frac{T}{V} \left[\frac{N_+^2}{\tilde{\kappa}_{++}} + \frac{N_+ N_-}{\tilde{\kappa}_{+-}} + \frac{N_- N_+}{\tilde{\kappa}_{-+}} + \frac{N_-^2}{\tilde{\kappa}_{--}} \right]. \qquad (5)$$

It is clear that κ_T reduces to the standard result for balanced populations where $N_- = 0$ and $N_+ = N$, $\kappa_T^{-1} = TV^{-1}N_+^2/\tilde{\kappa}_{++}$, leading to the standard form of the fluctuation-dissipation theorem: $\kappa_T = VT^{-1}\left[\langle \hat{N}^2 \rangle - \langle \hat{N} \rangle^2 \right] / \langle \hat{N} \rangle^2$. The connection between the two representations is simple. The first diagonal term is $\tilde{\kappa}_{++} = \tilde{\kappa}_{\uparrow\uparrow} + \tilde{\kappa}_{\downarrow\downarrow} + 2\tilde{\kappa}_{\uparrow\downarrow}$, the second diagonal term is $\tilde{\kappa}_{--} = \tilde{\kappa}_{\uparrow\uparrow} + \tilde{\kappa}_{\downarrow\downarrow} - 2\tilde{\kappa}_{\uparrow\downarrow}$, while the off-diagonal terms $\tilde{\kappa}_{+-} = \tilde{\kappa}_{-+} = \tilde{\kappa}_{\uparrow\uparrow} - \tilde{\kappa}_{\downarrow\downarrow}$ are identical by symmetry.

The pseudo-spin susceptibility χ_{zz} of the system can also be extracted from the pseudo-compressibility matrix $\tilde{\kappa}_{ij}$, since μ_- plays the role of an effective magnetic field h_z along the quantization axis z, and N_- plays the role of the magnetization m_z. For imbalanced Fermi systems this implies that

$$\tilde{\kappa}_{--} = T \frac{\partial N_-}{\partial \mu_-}\Big|_T = T \frac{\partial m_z}{\partial h_z}\Big|_T = T\chi_{zz}, \qquad (6)$$

and generalizes the results obtained for ultra-cold fermions interacting via p-wave interactions [9].

Compressibility matrix in a trap: In the presence of a trapping potential $V_\alpha(\mathbf{r})$, we define the local chemical potential as $\mu_\alpha(\mathbf{r}) = \mu_\alpha - V_\alpha(\mathbf{r})$, and rewrite the Hamiltonian explicitly as $H = H_1 - \int d\mathbf{r} \mu_\alpha(\mathbf{r}) \hat{n}_\alpha(\mathbf{r})$, where $\hat{n}_\alpha(\mathbf{r}) = \psi^\dagger(\mathbf{r})\psi(\mathbf{r})$ corresponds to the particle density operator. In this case, the grand partition function is $Z[T, V, \mu_\alpha(\mathbf{r})] = \mathrm{Tr} e^{-[H_1 - \int d\mathbf{r} \mu_\alpha \hat{n}_\alpha(\mathbf{r})]/T}$. The thermodynamic potential $\Omega[T, V, \mu_\alpha(\mathbf{r})]$ is also a functional of $\mu_\alpha(\mathbf{r})$, such that local and even non-local quantities can be extracted. For instance the local density $n_\alpha(\mathbf{r}) = \langle \hat{n}_\alpha(\mathbf{r}) \rangle$ is simply written as the functional derivative $Vn_\alpha(\mathbf{r}) = N_\alpha(\mathbf{r}) = -\delta\Omega/\delta\mu_\alpha(\mathbf{r})$, where $N_\alpha(\mathbf{r})$ is the local number of particles. Correspondingly the non-local pseudo-compressibility matrix is $\tilde{\kappa}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = -T\delta^2\Omega/\delta\mu_\alpha(\mathbf{r})\delta\mu_\beta(\mathbf{r}')$, which in terms of the local particle number operators $\hat{N}_\alpha(\mathbf{r})$ and $\hat{N}_\beta(\mathbf{r}')$ becomes

$$\tilde{\kappa}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \langle \hat{N}_\alpha(\mathbf{r}) \hat{N}_\beta(\mathbf{r}') \rangle - \langle \hat{N}_\alpha(\mathbf{r}) \rangle \langle \hat{N}_\beta(\mathbf{r}') \rangle. \qquad (7)$$

Since $N_\alpha(\mathbf{r}) = \langle \hat{N}_\alpha(\mathbf{r}) \rangle$ is a local thermodynamic average, we rewrite $\tilde{\kappa}_{\alpha\beta}(\mathbf{r}, \mathbf{r}') = \langle \delta\hat{N}_\alpha(\mathbf{r}) \delta\hat{N}_\beta(\mathbf{r}') \rangle$, where $\delta\hat{N}_\alpha(\mathbf{r}) = \hat{N}_\alpha(\mathbf{r}) - N_\alpha(\mathbf{r})$ is the local fluctuation in the number of particles of type α. The local pseudo-compressibility matrix is simply $\tilde{\kappa}_{\alpha\beta}(\mathbf{r}) = \tilde{\kappa}_{\alpha\beta}(\mathbf{r}, \mathbf{r})$.

Within the local density approximation (LDA) the local isothermal compressibility $\kappa_T(\mathbf{r})$ can be derived from the local pressure $P(\mathbf{r})$ as $\kappa_T(\mathbf{r}) = -V\partial P(\mathbf{r})/\partial V|_T$. Following the steps leading to Eq. (4), we obtain

$$\frac{1}{\kappa_T(\mathbf{r})} = \frac{T}{V} \sum_{\alpha\beta} \frac{1}{\kappa_{\alpha\beta}(\mathbf{r})}, \qquad (8)$$

which is the local generalization of the fluctuation-dissipation theorem within LDA. Here, $\kappa_{\alpha\beta}(\mathbf{r}) = \tilde{\kappa}_{\alpha\beta}(\mathbf{r}) / [N_\alpha(\mathbf{r}) N_\beta(\mathbf{r})]$.

Imbalanced Fermi-Fermi mixtures: As an example of the general relations just derived we discuss the case of imbalanced Fermi-Fermi mixtures with equal masses, which has attracted a lot of interest [10–15].

The resulting action corresponding to the Hamiltonian given in Eq. (1) has been succesfully calculated for the case of a uniform superfluid [16] in the Gaussian approximation as $S_G = S_0 + \frac{1}{2T}\sum_q \Lambda^\dagger(q)\mathbf{F}^{-1}\Lambda(q)$, where $q = (\mathbf{q}, \nu_\ell)$, with $\nu_\ell = 2\pi\ell T$ being the bosonic Matsubara frequeny at temperature T. Here, $\Lambda(q)$ is the order parameter fluctuation field and the matrix $\mathbf{F}^{-1}(q)$ is the inverse fluctuation propagator. Furthermore,

$$S_0 = \frac{|\Delta_0|^2}{gT} + \frac{1}{T}\sum_{\mathbf{k}}\left(\xi_{\mathbf{k},+} - E_{\mathbf{k},+}\right) + \sum_{\alpha}\ln\left[n_F(-E_{\mathbf{k},\alpha})\right],$$

is the saddle point action, where $E_{\mathbf{k},\alpha} = \sqrt{\xi^2_{\mathbf{k},+} + |\Delta_{\mathbf{k}}|^2} + s_\alpha\xi_{\mathbf{k},-}$ is the quasiparticle energy when $s_\uparrow = 1$ and is the negative of the quasihole energy when $s_\downarrow = -1$. We also use the notation $E_{\mathbf{k},\pm} = \left(E_{\mathbf{k},\uparrow} \pm E_{\mathbf{k},\downarrow}\right)/2$. In addition, $\Delta_{\mathbf{k}} = \Delta_0\Gamma_{\mathbf{k}}$ is the order parameter for superfluidity for pairing with zero center of mass momentum, $\Gamma_{\mathbf{k}} = 1$ for s-wave pairing, $n_F(E_{\mathbf{k},\alpha})$ is the Fermi distribution, and $\xi_{\mathbf{k},\pm} = \left(\xi_{\mathbf{k},\uparrow} \pm \xi_{\mathbf{k},\downarrow}\right)/2 = k^2/(2m_\pm) - \mu_\pm$, where $m_\pm = 2m_\uparrow m_\downarrow/\left(m_\downarrow \pm m_\uparrow\right)$. Notice that m_+ is twice the reduced mass of the $\uparrow$ and $\downarrow$ fermions, and that the equal mass case $(m_\uparrow = m_\downarrow)$ corresponds to $|m_-| \to \infty$.

The fluctuation term in the action leads to a correction to the thermodynamic potential, which can be written as $\Omega_G = \Omega_0 + \Omega_{\rm fluct}$, where $\Omega_0 = TS_0$ and $\Omega_{\rm fluct} = T\sum_q \ln det\left[T\mathbf{F}^{-1}(q)\right]$. The saddle point condition $\delta\Omega_0/\delta\Delta_0^* = 0$ leads to the order parameter equation

$$\frac{1}{g} = \sum_{\mathbf{k}}\frac{|\Gamma_{\mathbf{k}}|^2}{2E_{\mathbf{k},+}}X_{\mathbf{k},+}, \tag{9}$$

where $X_{\mathbf{k},\pm} = \left(X_{\mathbf{k},\uparrow} \pm X_{\mathbf{k},\downarrow}\right)/2$, with $X_{\mathbf{k},\alpha} = \tanh\left(E_{\mathbf{k},\alpha}/(2T)\right)$. As usual, we eliminate g in favor of the scattering length a_s via the relation $1/g = -m_+V/(4\pi a_s) + \sum_{\mathbf{k}}|\Gamma_{\mathbf{k}}|^2/(2\epsilon_{\mathbf{k},+})$, where $\epsilon_{\mathbf{k},\pm} = \left(\epsilon_{\mathbf{k},\uparrow} \pm \epsilon_{\mathbf{k},\downarrow}\right)/2 = k^2/(2m_\pm)$. The order parameter equation needs to be solved self-consistently with the number equations $N_\alpha = -\frac{\partial\Omega}{\partial\mu_\alpha}\Big|_T$, which has two contributions

$$N_\alpha = N_{0,\alpha} + N_{{\rm fluct},\alpha}, \tag{10}$$

where $N_{0,\alpha} = -\frac{\partial\Omega_0}{\partial\mu_\alpha}\Big|_T$ is the saddle point number equation given by

$$N_{0,\alpha} = \sum_{\mathbf{k}}\left(\frac{1 - s_\alpha X_{\mathbf{k},-}}{2} - \frac{\xi_{\mathbf{k},+}}{2E_{\mathbf{k},+}}X_{\mathbf{k},+}\right) \tag{11}$$

and $N_{{\rm fluct},\alpha} = -\partial\Omega_{\rm fluct}/\partial\mu_\alpha|_T$ is the fluctuation contribution to N_α given by $N_{{\rm fluct},\alpha} = -T\sum_q\{\partial\left[det\mathbf{F}^{-1}(q)\right]/\partial\mu_\alpha\}/det\mathbf{F}^{-1}(q)$.

To calculate $\tilde\kappa_{\alpha\beta}$ and κ_T from the thermodynamic potential Ω, we note that $\Omega\left[\Delta_0(\mu_\uparrow,\mu_\downarrow),\mu_\uparrow,\mu_\downarrow,T\right] \to \Omega\left[\mu_\uparrow,\mu_\downarrow,T\right]$. Because of the implicit dependence of Δ_0 on $\mu_\uparrow$ and $\mu_\downarrow$, the calculation of $\tilde\kappa_{\alpha\beta}$ requires

$$\tilde\kappa_{\alpha\beta} = T\frac{\partial N_\alpha}{\partial\mu_\beta}\Big|_{T,e} + T\frac{\partial N_\alpha}{\partial|\Delta_0|^2}\Big|_{T,e}\cdot\frac{\partial|\Delta_0|^2}{\partial\mu_\beta}\Big|_{T,i}, \tag{12}$$

where the label "e" ("i") means explicit (implicit) derivative. The mechanical stability of the uniform superfluid and normal phases is guaranteed if all eigenvalues of $\tilde\kappa_{\alpha\beta}$ (or $\tilde\kappa_{ij}$) are positive. This is achieved when ${\rm Tr}\tilde\kappa > 0$ and $\det\tilde\kappa > 0$. When the lowest eigenvalue of $\tilde\kappa$ reaches zero then the system becomes mechanically unstable.

We define the Fermi momentum $k^3_{F+} = k^3_{F\uparrow} + k^3_{F\downarrow}$, where $k_{F\alpha}$ is the Fermi momentum of each species, and the Fermi energy $\epsilon_{F+} = k^2_{F+}/(2m_+)$. For a mixture of fermions of equal masses, different hyperfine states and no trapping potential, the zero temperature phase diagram of population imbalance $P = (N_\uparrow - N_\downarrow)/(N_\uparrow + N_\downarrow) = N_-/N_+$ versus scattering parameter $1/(k_{F+}a_s)$ is shown in Fig. 1a, where the normal (N), non-uniform (NU) and uniform (U) superfluid regions are indicated. The isothermal compressibility κ_T is shown in Fig. 1b for $1/(k_{F_+}a_s) = 2.16$ and changing P. Notice that as P increases, the dimensionless compressibility $\kappa_T\epsilon_{F_+}/V$ diverges at a critical population imbalance $P_c = 0.77$, and becomes negative for $P > P_c$ signaling a quantum phase transition from uniform superfluidity with coexistence of excess unbound fermions and paired fermions in the same spatial region to a phase separated regime where excess unbound fermions and paired fermions tend to avoid being in the same region of space.

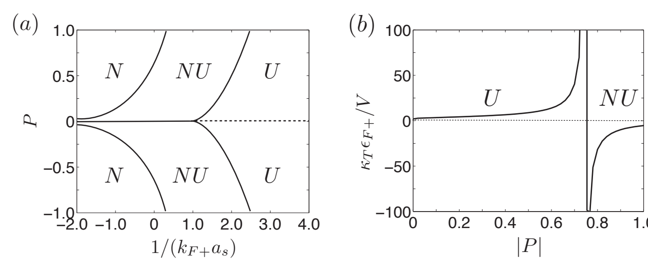

FIG. 1: a) The zero temperature phase diagram of population imbalance $P = N_-/N_+$ versus scattering parameter $1/(k_{F+}a_s)$ for equal mass fermions is shown. b) The dimensionless isothermal compressibility $\kappa_T\epsilon_{F_+}/V$ is shown for fixed $1/(k_{F+}a_s) = 2.16$.

In Fig. 2a, we show the dimensionless matrix elements $\tilde\kappa_{\alpha\beta}\epsilon_{F+}/T$ as a function of $1/(k_{F_+}a_s)$ on the BEC side $[1/(k_{F_+}a_s) > 0]$ for fixed population imbalance $P = 0.5$. Notice that $\tilde\kappa_{\alpha\beta}$ diverges as $(\lambda - \lambda_c)^{-1}$, where $\lambda = 1/(k_{F_+}a_s)$, and $\lambda_c = 1.9$ is the critical interaction parameter. As seen in Fig. 2b, when population imbalance is changed in the BEC regime, e.g. $1/(k_{F_+}a_s) = 2.16$, the dimensionless pseudo-spin susceptibility $\chi_{zz}\epsilon_{F+} = \tilde\kappa_{--}\epsilon_{F+}/T$ diverges at the phase

boundary between the uniform superfluid and the non-uniform phases as $\chi_{zz}\epsilon_{F+} \sim (P - P_c)^{-1}$. The negative values of the matrix elements $\tilde{\kappa}_{\alpha\beta}\epsilon_{F+}/T$ and $\chi_{zz}\epsilon_{F+}$ just indicate the region of non-uniform superfluidity, i.e., the region where uniform superfluidity is not mechanically stable. Thus, the generalized fluctuation-dissipation theorem described in Eq. (3) allows for the extraction of critical exponents of density-density and pseudospin-pseudospin correlations accross phase boundaries.

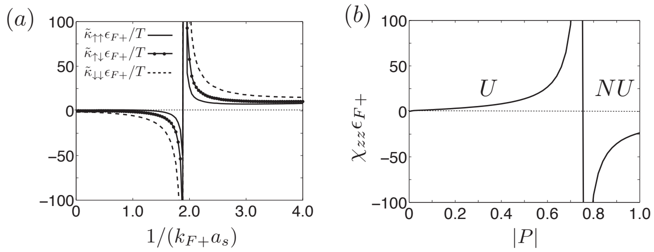

FIG. 2: a) Pseudo-compressibility matrix elements $\tilde{\kappa}_{\alpha\beta}\epsilon_{F+}/T$ as a function of $1/(k_{F+}a_s)$ for population imbalance $P = 0.5$; b) Pseudo-spin susceptibility $\chi_{zz}\epsilon_{F+} = \tilde{\kappa}_{--}\epsilon_{F+}/T$ as a function of P for interaction parameter $1/(k_{F+}a_s) = 2.16$.

In the case of a non-zero trapping potential $V_\alpha(\mathbf{r})$, we make use of LDA and obtain the local thermodynamic potential $\Omega(\mathbf{r}) = \Omega\left[\mu_\uparrow(\mathbf{r}), \mu_\downarrow(\mathbf{r})\right]$ from the thermodynamic potential in the absence of a trap $\Omega\left[\mu_\uparrow, \mu_\downarrow\right]$, via the substitution $\mu_\alpha \to \mu_\alpha(\mathbf{r}) = \mu_\alpha - V_\alpha(\mathbf{r})$. This implies that in Eqs. (9) and (10) the order parameter Δ_0 and the number of particles N_α become functions of position $\mathbf{r}$ via the position dependent chemical potentials $\mu_\alpha(\mathbf{r})$. As a result we have $\Delta_0(\mathbf{r}) = \Delta_0\left[\mu_\uparrow(\mathbf{r}), \mu_\downarrow(\mathbf{r})\right]$ and $N_\alpha(\mathbf{r}) = N_\alpha\left[\mu_\uparrow(\mathbf{r}), \mu_\downarrow(\mathbf{r})\right]$.

We consider harmonic trapping potentials $V_\alpha(\mathbf{r}) = \gamma_\alpha r^2/2$, where $\gamma_\alpha = m_\alpha\omega_\alpha$, with ω_α being the trapping frequencies of fermion of type α. For the equal mass case, we show in Fig. 3a, the particle number profiles $N_\alpha(\mathbf{r})$ and the order parameter $\Delta_0(\mathbf{r})$ as a function of dimensionless position $\mathbf{r}/r_{TF}$, where r_{TF} is the Thomas-Fermi radius defined through the condition $\epsilon_{F+} = \gamma_+ r_{TF}^2/2$, where $\gamma_+ = \gamma_\uparrow + \gamma_\downarrow$. These spatial profiles show that superfluidity coexists with excess unpaired fermions, but the majority of excess unpaired fermions are pushed away from the center of the trap. In Fig. 3b, we show the spatial dependence of $\tilde{\kappa}_{\alpha\beta}(\mathbf{r})$, from which the local correlation functions $\langle \hat{N}_\alpha(\mathbf{r})\hat{N}_\beta(\mathbf{r})\rangle$ and the pseudo-spin susceptibility $\chi_{zz}(\mathbf{r}) = \tilde{\kappa}_{--}/T$ can also be easily extracted. Within LDA, $\tilde{\kappa}_{\alpha\beta}(\mathbf{r})$ exhibit a discontinuous jump at the position $\mathbf{r}_c$ where $\Delta_0(\mathbf{r}_c) = 0$. In the superfluid region, local particle fluctuations reveal the extra correlations brought in by full pairing such that $\chi_{zz}(\mathbf{r}) = 0$. Outside the superfluid region local particle fluctuations show a decrease in particle-particle correlations, as the excess unpaired fermions are pushed away from the center of the trap, leading to $\chi_{zz}(\mathbf{r}) \neq 0$.

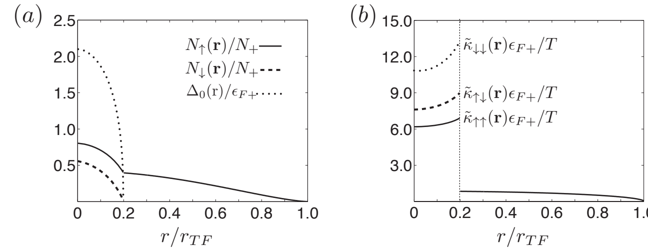

FIG. 3: a) Particle number profiles $N_\alpha(\mathbf{r})/N_+$, and order parameter $\Delta_0(\mathbf{r})/\epsilon_{F+}$; b) Matrix elements $\tilde{\kappa}_{\alpha\beta}\epsilon_{F+}/T$ as a function of $\mathbf{r}/r_{TF}$, for population imbalance $P = 0.6$ and interaction parameter $1/(k_{F+}a_s) = 3.0$.

Summary: We derived a generalized fluctuation-dissipation theorem for Fermi-Fermi mixtures, which was used to extract thermodynamic information (compressibility, spin-susceptibility, phase diagrams and critical exponents) from density and density-fluctuation profiles of imbalanced mixtures of equal or unequal masses. We discussed continuum systems with and without trapping potentials. Using the local density approximation, we obtained expressions relating the local compressibility and local spin susceptibility to the local fluctuations in particle numbers. Lastly, we applied our results to the case of population imbalanced Fermi systems of equal masses.

We would like to thank the Army Research Office (Contract No. W911NF-09-1-0220) for support.

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